
ASSESSMENTS OF CONTINUOUS SAMPLING PLAN INDEXED THROUGH MAXIMUM ALLOWABLE AVERAGE OUTGOING QUALITY

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ABSTRACT

This paper introduces a result oriented designing and selection procedure for Continuous Sampling Plan type of CSP-M indexed through Maximum Allowable Average Outgoing Quality (MAAOQ) for final products. Expressions for performance measures such as Operating Characteristic function and MAAOQ function are derived. It is a preferred quality index for engineers and quality controllers.

Keywords: Acceptable Quality Level, Average Outgoing Quality Level, Maximum Allowable Average Outgoing Quality, Maximum Allowable Percent Defective.

INTRODUCTION

Continuous sampling plans (CSPs) are used in continuous production processes where no separate lots are formed for inspection. They are generally used on some type of conveyor but are applicable to any continuous type operation where it is not desirable to accumulate the product into lots for purposes of inspection. Dodge (1943) has introduced the concept of CSP and provided mathematical rationale and rules of operation for CSP-1. Extensions of CSP-1 which have received considerable attention

and use are devised by Dodge and Torrey (1951) and labeled as CSP-2 and CSP-3. As indicated in their paper, both plans grew out of suggestions given by inspection personnel engaged in applying continuous sampling.

CSPs have an acceptance number 'c', which is greater than zero, while inspection. CSP-1 has two parameters 'i' and 'f'. 'i' is a clearance number and 'f' is a sampling frequency. In CSP-2 an additional parameter 'k' is introduced which is a minimum number of consecutive sampled units. That is, it allows for sampling to continue with

the occurrence of an occasional defect provided that a defect does not occur too frequently. CSP-3 introduces a simple and effective refinement of CSP-2 designed to provide extra protection against the case of spotty quality, i.e., the clustering of excessive defectiveness. The additional four consecutive units are inspected upon finding a defect in sampling on the CSP-2 basis. If the given conditions are not up to the satisfactory level, the 100% inspection phase is invoked immediately. Evaluation of CSP-2 and CSP-3 is similar to that of CSP-1. Only slight variations in some symbols and definitions are presented. The other conditions are the same as for CSP-1. Chung-Ho Chen (2004) developed the AOQL for lot by lot CSPs, which is one of the indices to measure the performance of the CSP-1.

Suresh and Ramkumar (1996) have proposed a new procedure for the selection of a Single Sampling Plan (SSP) in terms of the AOQL and MAAOQ. The AOQL is defined as the poorest average quality that the consumer will receive in the long run, when defective items are replaced by non-defective items. Mandelson (1962) has explained the desirability of developing a system of sampling plans indexed through

the MAPD and shown that $p^* = c/n$. Suresh and Nirmala (2015) have presented construction and selection of various CSPs indexed through quality decision regions and studied comparison of certain type of CSPs and their operating procedures. Many works on designing of continuous sampling plans are available in the literature, which are proposed by various authors.

GLOSSARY OF SYMBOLS:

p = probability of a unit produced by the process being non-conforming

q = $1 - p$

i = clearance number

f = sampling frequency

F = Average Fraction Inspected

P_* = MAPD = Maximum Allowable Percent Defective

p_m = AOQL = Average Outgoing Quality Level

$AOQ(p)$ = Average Outgoing Quality when the process non-conformance Probability is p

P_{MAOQ} = MAAOQ = Maximum Allowable Average Outgoing Quality

$Pa(p)$ = probability of acceptance during sampling phase when the

process non-conformance probability is p . Technical terms are defined as in ANSI (1987) standards.

MAXIMUM ALLOWABLE PERCENT DEFECTIVE (MAPD)

The point on the OC curve at which the descent is steepest is called point of inflection. The proportion nonconforming corresponding to the point of inflection of OC curve is interpreted as the maximum allowable percent defective.

MAXIMUM ALLOWABLE AVERAGE OUTGOING QUALITY (MAAOQ)

The MAAOQ of a sampling plan is designated as the Average Outgoing Quality (AOQ) at the MAPD.

$$AOQ = p P_a(p)$$

Then MAAOQ = AOQ at $p = p_*$ which can be rewritten as,

$$MAAOQ = p_* * P_a(p_*)$$

One of the desirable properties of an OC curve is that the decrease of $P_a(p)$ should be lower for smaller values of p and steeper for higher values of p , which provides better overall discrimination. Since p corresponds to the inflection point of an OC curve, it implies that

$$\frac{d^2 P_a(p)}{dp^2} < 0, \quad \text{for } p < p_*$$

$$\frac{d^2 P_a(p)}{dp^2} > 0, \quad \text{for } p > p_*$$

$$\frac{d^2 P_a(p)}{dp^2} = 0, \quad \text{for } p = p_*$$

CONNECTIVITY OF AOQL, MAPD AND MAAOQ IN CSPS PLAN

The collection of average outgoing quality limit (AOQL), Maximum allowable present defective (MAPD) and maximum allowable average outgoing quality (MAAOQ) for all CSPs plans are consider. It is found that MAAOQ is more precious than AOQL. For example, Table are presented for designing of all types of CSPs plans, for given consideration. Here MAAOQ value is very less compared to AOQL value. It implies that through MAAOQ designing, additional strengthening is ensured to the producer from the risk of rejecting the good quality items measure up to AOQL designing, which is perceptible through figure 1.

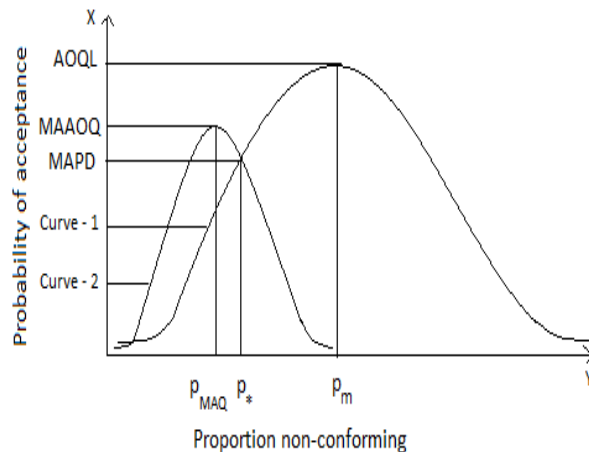


Figure: 1: The AOQL and MAAOQ curves for CSP plan

Figure 1 in attendance the Average Outgoing Quality (AOQ) curves along with AOQL curve and MAAOQ curve. It can be observed that two curves are in desirable shape with a engorge at higher value of AOQL in curve-1 and a sudden drop at higher value of MAAOQ in curve-2. It point towards that the MAAOQ practice doubly protects the security of the consumer in terms of incoming and outgoing quality and protects the producer's interest against acceptable quality and safeguards the consumer against poor quality.

Thus, the addition of prior information about the process the past benefits the producer with minimising the risk for discarded items of good quality. However, the CSP satisfies the interests of the consumer by accepting poor quality

items with probabilities not exceeding the assigned risk of non-conformities. On the other hand, the nature of the AOQ curve-2 of the CSP provides relatively more protection to the consumer against the items of poor quality.

The procedure for designing a CSP with quality standards p^* and MAAOQ, where p^* is the quality standard to the Y-axis from the inflection point of the AOQ curve-2. The MAAOQ of a CSP is defined as the AOQ at the MAPD. Using MAPD as a standard, and the MAAOQ as an average outgoing quality, the parameters for CSP are determined. The AQLs for the plans are also provided for a fixed producer risk ($\alpha = 0.05$). The Technical terms are defined as in ANSI (1987) standards.

DESIGNING PLANS FOR GIVEN MAPD

The proportion nonconforming corresponding to the inflection point of the OC curve, denoted by p^* and interpreted as the Maximum Allowable Percent Defective (MAPD) by Mayer (1967) is also used as the quality standard along with some other conditions for the selection of the sampling

plans. The relative slope of the OC curve at this point, denoted as h^* is also used to fix the discrimination of the OC curve of any sampling plan. The desirability of developing a set of sampling plans indexed by p^* has been explained by Mandelson (1962) and Soundararajan (1971). While choosing a plan for given p^* , one has to specify the measure of discrimination $K = pT / p^*$, where pT is the point at which the tangent line at the inflection point of the OC curve cuts the p -axis or h^* , the relative slope of the OC curve at p^* . Suresh and Srivenkataramana (1996) have designed procedure for the selection of single sampling plan using producer and consumer quality levels. Suresh (1993) has studied various sampling plans with the quality levels along with their relative slopes.

According to Suresh and Ramkumar (1996) the Maximum Allowable Average Outgoing Quality is the outgoing quality defined with p which is a favoured quality index for engineers and it protects the interests of the consumer. Considering the simplicity, practicability and consumer protection offered, the MAAOQ has major practical advantages in acceptance sampling compared with AOQL, which can be considered as a measure for selection of plan

parameters. Dodge and Romig (1959) have proposed procedure for the selection of Single Sampling Plan indexed through AOQL by minimizing the Average Total Inspection. Soundararajan (1981) has suggested procedure for the selection of Single Sampling Plan in terms of AQL and AOQL.

FOR SPECIFIED MAAOQ AND MAPD

Table 1 is used to construct the plans when MAPD and MAAOQ are specified. For any given values of MAPD (p_*) and MAAOQ (p_{MAOQ}), find the value in Table 1 under the column R_1 which is approximately equal to the calculated value. Then the corresponding value of c and f are noted. From this one can determine the parameters c and f for the continuous sampling plan-M.

FOR SPECIFIED AOQL AND MAPD

Table 1 is used to construct continuous sampling plan-M for given MAPD and MAAOQ quality levels. For any given values of the i and f one can find the performance measure MAAOQ and various ratios $R_1 = MAAOQ / MAPD$, $R_2 = AOQL / MAPD$ and $R_3 = AQL / MAPD$.

NUMERICAL EXAMPLES

1. Given MAPD = 0.65241, and MAAOQ = 0.00272 compute the ratio R_1 which is $R_1 = \text{MAAOQ} / \text{MAPD} = 0.00272 / 0.65241 = 0.00417$, $R_2 = \text{AOQL} / \text{MAPD} = 0.46387 / 0.65241 = 0.71101$ and $R_3 = \text{AQL} / \text{MAPD} = 0.05096 / 0.65241 = 0.07812$ which is associated with $c = 2$, $f = 1/2$. Thus $c = 2$, $f = 1/2$ are the parameters selected for continuous sampling plan CSP – M for a given MAPD of 0.65241 and MAAOQ of 0.00272 defective.

CONVERSION OF PARAMETERS

Table 1 may be used to convert continuous sampling plan-M from one set of parameters to other familiar sets, which will provide related information on the derived plan. For example, given AOQL = 0.73932 and MAAOQ = 0.08286. The value corresponding to this ratios in MAAOQ / MAPD = 0.08286 / 0.73612 = 0.11256, AOQL / MAPD = 0.73932 / 0.73612 = 1.00435 and AQL / MAPD = 0.65639 / 0.73612 = 0.8917.

CONSTRUCTION OF TABLES

The expression for the OC function of continuous sampling plan-M is given by,

$$P_a(p) = \frac{q^i (f^k + fk)}{f^{k+1} + q^i (f^k + fk - f^{k+1})}$$

$$R_1 = \text{MAAOQ} / \text{MAPD}, R_2 = \text{AOQL} / \text{MAPD} \& R_3 = \text{AQL} / \text{MAPD}$$

The incoming quality MAPD column of Table 1 is constructed by equation 1 is equating the second order derivative of operating characteristic function. The values of AQL, AOQL, MAAOQ, MAPD, R1, R2 and R3 are given.

CONCLUSION

The present work mainly emphasizes that the selection of sampling system with this procedure is more advantageous to the producer and consumer than conventional methods. This method facilitates user friendly attitude for engineers who are working on the floor and the system that

adopt readymade tables, which are provided for selecting plans based on specific input/output parameters. This design of sampling system is constructed for application towards industrial shop-floor situations for manufacturing of product/process, which are tailor made to the industrial situations. The main advantages of this technique are less handling of units

during inspection, more Economical owing to fewer inspections thereby simplifying recruiting, training and supervising.

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TABLE 1: VALUES OF QDR, PQR, IQR AND LQR FOR SPECIFIED VALUES OF ‘T’ AND ‘F’ FOR CONTINUOUS SAMPLING PLAN (CSP-M)

i	f	nd ₁	nd ₂	nd ₀	nd ₃	T	T ₁	T ₂
2	1/2	0.52957	0.6779	0.4832	0.1483	0.7811	3.569	1.09597
			3	0	6	6	6	
2	1/3	0.53798	0.6798	0.4764	0.1418	0.7913	3.791	1.12917
			6	4	8	1	9	
2	1/4	0.63037	0.7621	0.5362	0.1317	0.8271	4.785	1.17563
			1	1	4	4	1	
3	1/5	0.39972	0.5585	0.3970	0.1588	0.7156	2.517	1.00671
			3	6	1	7	0	
3	1/6	0.41829	0.5783	0.4173	0.1600	0.7233	2.614	1.00233
			1	2	2	0	1	
3	1/7	0.41375	0.5680	0.3985	0.1542	0.7284	2.682	1.03828
			1	1	6	3	2	
3	1/8	0.43465	0.5896	0.4205	0.1550	0.7370	2.803	1.03361
			9	2	4	9	5	

3	1/9	0.44128	0.5820	0.3877	0.1407	0.7581	3.135	1.13806
			4	5	6	7	1	
3	2/3	0.46497	0.6062	0.4120	0.1412	0.7669	3.291	1.12855
			3	1	6	9	7	
4	2/4	0.30612	0.4634	0.3110	0.1573	0.6605	1.945	0.98419
			4	4	2	5	9	
4	2/5	0.31877	0.4774	0.3255	0.1586	0.6677	2.009	0.97912
			0	7	3	3	6	
4	2/6	0.32687	0.4863	0.3347	0.1595	0.6720	2.049	0.97638
			7	8	0	7	4	
4	2/7	0.32165	0.4760	0.3172	0.1543	0.6757	2.084	1.01394
			1	3	5	4	0	
4	2/8	0.33562	0.4909	0.3326	0.1553	0.6836	2.160	1.00909
			6	0	4	1	6	
4	2/9	0.34569	0.5010	0.3428	0.1553	0.6899	2.225	1.00829
			2	5	3	8	6	
4	3/4	0.30527	0.5018	0.3247	0.1965	0.6083	1.553	0.93997
			3	7	6	2	1	
4	3/5	0.37364	0.5185	0.3416	0.1449	0.7205	2.578	1.09367
			5	4	1	5	5	
4	3/6	0.38656	0.5313	0.3545	0.1447	0.7275	2.670	1.09045
			4	0	8	2	1	
5	3/7	0.23797	0.3905	0.2370	0.1525	0.6093	1.559	1.00385
			4	6	7	4	8	
5	3/8	0.24685	0.4006	0.2476	0.1537	0.6161	1.605	0.99662
			1	9	6	9	5	
5	3/9	0.25321	0.4078	0.2551	0.1546	0.6208	1.637	0.99233
			1	6	1	8	7	
5	4/5	0.25880	0.4129	0.2604	0.1541	0.6267	1.679	0.99368
			2	5	2	6	3	

5	4/6	0.25329	0.4039 0	0.2455 0	0.1506 1	0.6271 2	1.681 8	1.03174
5	4/7	0.26301	0.4146 2	0.2566 4	0.1516 1	0.6343 5	1.734 8	1.02483
5	4/8	0.27038	0.4225 1	0.2647 3	0.1521 2	0.6399 6	1.777 5	1.02135
5	4/9	0.27651	0.4283 3	0.2706 8	0.1518 2	0.6455 6	1.821 4	1.02155
5	5/6	0.29253	0.4360 1	0.2630 3	0.1434 7	0.6709 5	2.039 0	1.11217
5	5/7	0.30341	0.4471 2	0.2743 3	0.1437 1	0.6785 9	2.111 3	1.10601
5	5/8	0.31264	0.4564 8	0.2837 5	0.1438 4	0.6849 0	2.173 6	1.10183
5	5/9	0.32039	0.4643 2	0.2916 3	0.1439 3	0.6900 3	2.226 1	1.09863

**Table 2: Certain Characteristic values for R_1 and R_2 (QDR /PQR and LQL/AQL)
Continuous Sampling Plan (CSP-M)**

i	f	p*	p₀	p₁	p₂	R₁	R₂
2	1/2	0.58548	0.53911	0.05591	0.73384	0.92081	13.1254
2	1/3	0.60862	0.54708	0.07064	0.75050	0.89888	10.6243
2	1/4	0.66030	0.56613	0.02993	0.79204	0.85738	26.4631
3	1/5	0.44178	0.43912	0.04206	0.60059	0.99397	14.2794
3	1/6	0.44005	0.43908	0.02176	0.60007	0.99779	27.5767
3	1/7	0.46681	0.45156	0.05306	0.62107	0.96733	11.7051
3	1/8	0.46566	0.45153	0.03101	0.62071	0.96965	20.0161
3	1/9	0.53584	0.48231	0.09456	0.67660	0.90010	7.15525

3	2/3	0.53522	0.48226	0.07025	0.67648	0.90104	9.62961
4	2/4	0.33809	0.34301	0.03197	0.49541	1.01454	15.4961
4	2/5	0.33613	0.34293	0.01736	0.49476	1.02022	28.5001
4	2/6	0.33499	0.34291	0.00812	0.49449	1.02360	60.8978
4	2/7	0.36216	0.35774	0.04051	0.51651	0.98779	12.7502
4	2/8	0.36070	0.35768	0.02508	0.51604	0.99162	20.5758
4	2/9	0.36053	0.35769	0.01484	0.51586	0.99211	34.7615
4	3/4	0.37902	0.39852	0.07375	0.57558	1.05144	7.80447
4	3/5	0.43049	0.39849	0.05685	0.57541	0.92566	10.1214
4	3/6	0.43056	0.39850	0.04401	0.57534	0.92553	13.0759
5	3/7	0.26236	0.26145	0.02439	0.41493	0.99652	17.0123
5	3/8	0.26051	0.26135	0.01366	0.41427	1.00321	30.3272
5	3/9	0.25934	0.26131	0.00614	0.41395	1.00755	67.4186
5	4/5	0.25972	0.26137	0.00092	0.41384	1.00634	449.826
5	4/6	0.28471	0.27691	0.03141	0.43531	0.97263	13.8590
5	4/7	0.28320	0.27683	0.02019	0.43481	0.97750	21.5359
5	4/8	0.28246	0.27681	0.01208	0.43458	0.97999	35.9752
5	4/9	0.28268	0.27685	0.00617	0.43450	0.97937	70.4214
5	5/6	0.35024	0.32074	0.05771	0.49371	0.91576	8.55502
5	5/7	0.34981	0.32072	0.04639	0.49351	0.91686	10.6383
5	5/8	0.34960	0.32071	0.03696	0.49344	0.91735	13.3506
5	5/9	0.34947	0.32071	0.02908	0.49340	0.91770	16.9670